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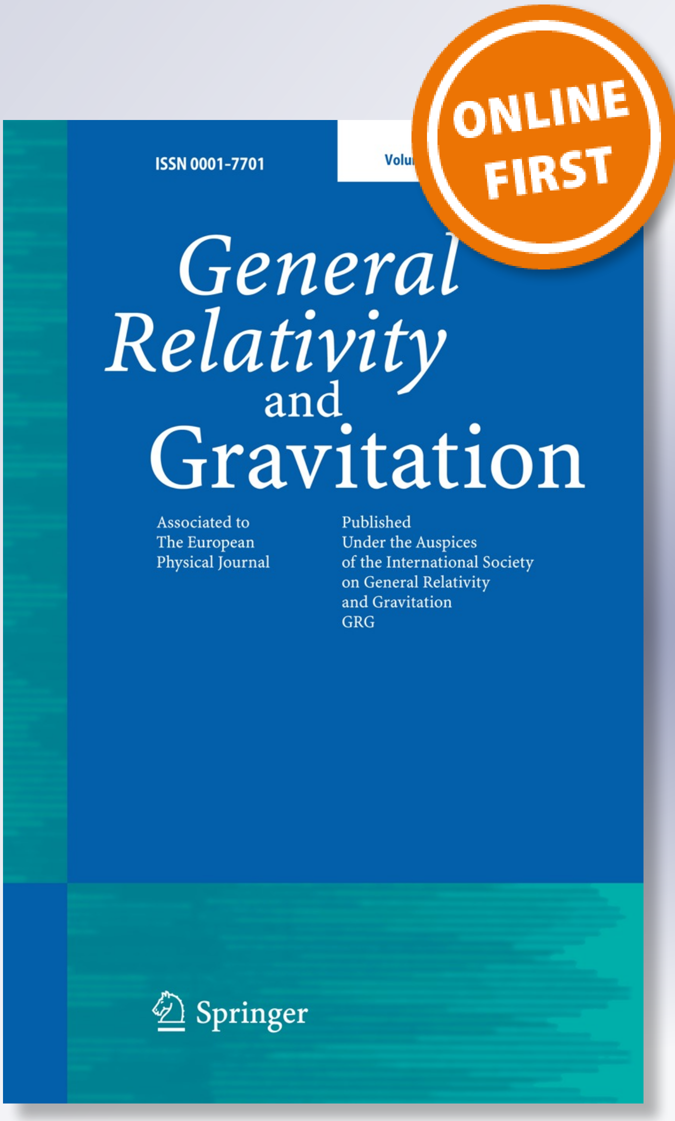

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RESEARCH ARTICLE

Test of the law of gravitation at small accelerations

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Abstract Newton's Law of Gravitation has been tested at small values a of the acceleration, down to $a \approx 10^{-10}$ m s^{-2}, the approximate value of MOND's constant a_0. Within experimental errors no deviations from Newton's Law were found. A comparison with six versions of the MOND interpolation function is given. Under the assumptions made in this paper one of the versions can be excluded.

Keywords Gravitation · Dark matter · MOND · Experimental results

1 Introduction

The nature of dark matter is one of the central questions in astrophysics at present. Introduced originally to explain the dynamics of galaxies, dark matter has found an established place in the Cosmological Model. Still, many questions and difficulties

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remain, see e.g. [1]. In this context, also alternative explanations are discussed, one of them being MOND (modified-Newtonian-dynamics) [2]. MOND assumes, that the gravitational law is modified for small values of the acceleration in the following way:

$$a_N = a \quad \mu(a/a_0) \tag{1}$$

Here, a is the acceleration according to MOND, a_N is the Newtonian acceleration $a_N = Gm/r^2$, and $a_0 = 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ is assumed to be a universal constant [1,3]. The interpolation function $\mu(a/a_0)$ is $\mu \to 1$ for $a \gg a_0$, recovering Newton's Law, and $\mu \to a/a_0$ for $a \ll a_0$. Apart from these asymptotic values the interpolation function is not determined by the theory, but has to be constrained by data. A relativistic formulation incorporating the MOND theory has been developed by Bekenstein [4].

MOND has so far passed many astronomical tests [1,3]. Apart from modifying Newton's Law, MOND could also be interpreted as a violation of Newton's second axiom $F = ma$ [5], irrespective of the nature of the force F. This latter aspect has been experimentally checked, using electromagnetic restoring forces, and Newton's axiom verified down to accelerations of $3 \times 10^{-11}\,\mathrm{m\,s^{-2}}$ [6] and $5 \times 10^{-14}\,\mathrm{m\,s^{-2}}$ [7]. Therefore, a possible modification according to MOND must rest with the gravitational force alone. An experiment relying only on the gravitational force, also for the restoring force, may therefore yield a more stringent test. This experiment is designed to test Newton's Law at accelerations of the order a_0, using only gravitational forces.

An experiment with a similar method [8] has given a very precise measurement of Newton's constant, but using accelerations exceeding a_0 by more than two orders of magnitude. It is therefore not sensitive to deviations from gravity according to MOND.

It has been argued, that such a laboratory test as presented here is not meaningful in the strong gravitational field of the earth, but, due to a lack of a deeper understanding of MOND, this view is not shared by everybody (see e.g. [3]).

It has also been pointed out, that a test of MOND may only be possible in a system, whose acceleration with respect to the center of the Galaxy is smaller than a_0 [9,10]. In this approach, it is assumed that Newton's second axiom is violated. Under this assumption the present experiment would not detect a deviation from standard gravitation.

Proposals for the ultimate test of MOND would require very accurate measurements of gravity at a saddle point of the earth-moon-sun system with the LISA pathfinder [11] or at a saddle point of the sun-Jupiter system [12].

2 Experimental procedure

A schematic view of the experiment is shown in Fig. 1. The central part of the experiment is a microwave resonator, tuned to a frequency of about 21.3905 GHz. The resonator consists of two mirrors with spherical surfaces, suspended by tungsten wires of about 3 m length, resulting in a pendulum period of 3.289 ± 0.010 s. Their motion is damped by a set of permanent magnets, forming an eddy current brake. This part of the detector sits in an evacuated vessel. Two field-masses are positioned outside the vacuum vessel on either side of the resonator, and are periodically and simulta-

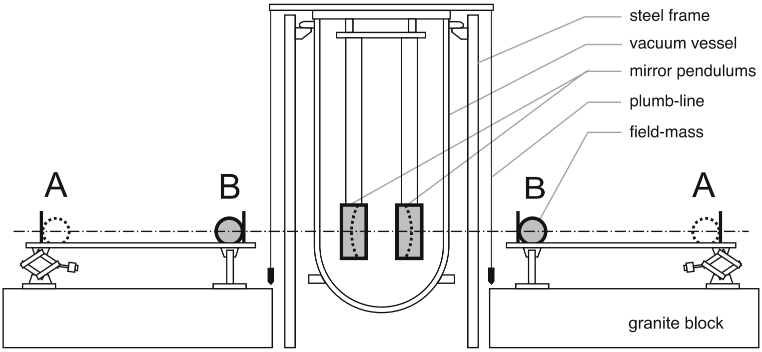

Fig. 1 Schematic view of the experiment. The damping magnets mounted underneath the mirror pendulums are not shown

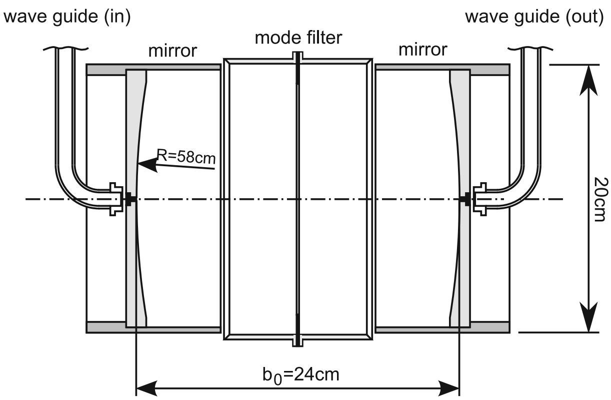

Fig. 2 Schematic view of the resonator

neously moved between a far (A) and a near (B) position. The resulting change of their gravitational pull leads to a small change of the position of the two mirrors, which is measured from the change of the resonance frequency.

A detailed view of the resonator is presented in Fig. 2, showing also the microwave guides.

The apparatus had been built and operated at Wuppertal University for a precision measurement of the gravitational constant [13–18]. It was later transferred to DESY and reinstalled with some improvements for the stability of the support [19,20].

Measurements were carried out with three pairs of field-masses, consisting of spheres of brass, marble and plastic, with masses of 9.02, 2.92 and 1.00 kg, respectively. All spheres have the same diameter of 12.7 cm. During the measurements the positions of the left (right) field-masses were automatically altered every 40 min between the near position at 76.6 cm (77.9 cm) and the far position at 213 cm (220 cm).[1]

[1] Distances are quoted between center of the spheres and center of gravity of the nearest mirror. Due to asymmetric placement of the cavity inside the vacuum vessel the distances on left and right side are not identical.

The acceleration of the left mirror caused by the closer field-mass at the near position was $10.3 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, $3.3 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, and $1.1 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ for the three masses, resulting in a change of the distance between the two mirrors ranging from about 0.210–0.023 nm. If Newton's Law is correct, the deflections due to the movement of the three field-masses must be precisely proportional to their mass values.

Measurements of the resonance frequency f_R were performed every two seconds by tuning the frequency of the generator to five values around the resonance frequency and recording the resulting amplitude at the exit of the resonator. The resonance frequency was then determined by fitting a resonance curve of the form Eq. (2) to the five amplitude values $U(f)$.

$$U(f) = U_{max} \frac{1}{1 + 4((f - f_R)/f_w)^2} \tag{2}$$

Here $U(f)$ is the amplitude at frequency f, f_R is the fitted resonance frequency and f_w is the resonance width.

The frequency measurements were then averaged over typically 1–2 min. The temperature at the apparatus was kept constant to about 0.1°; still the data show a strong drift with temperature, which must be corrected for.

Figure 3 shows an example of a measurement with the 9.02 kg field-masses before and after subtraction of a low frequency drift mainly due to temperature changes. In the upper part of the figure a large constant frequency offset has been subtracted from the measurement values.

Additional short-term distortions come from sources like ground movements, occasional earthquakes and waves from the North Sea: Fig. 4 shows as an example the rms noise of single frequency measurements plotted against the significant wave height at the mouth of the Elbe river.

3 Results

The data were evaluated using six different methods of background subtraction, to deal with slow drifts of the resonance frequency and with short-term background variations: Overall polynomial fits (A1, A3), a piecewise 3rd order polynomial fit (A5), a piecewise 5th order polynomial fit (A6), and sliding average (A2, A4) [20]. For each mass the uncertainties of the mean values in Table 1 were determined from the variance of the results of about 25 independent data runs with an average duration of 12 h. The differences between the results of different methods for the 9.02 kg data are somewhat larger than expected from the individual errors and reflect differences in dealing with short-term variations of the background. To take this into account, the uncertainty of the 9.02 kg data in Fig. 5 and in the calculation of G was increased by a factor of 1.5, according to the prescription of the Particle Data Group [21]. This prescription deals with the problem of calculating the mean and error of different measurements, which differ by more than their individual uncertainties. All six methods were checked with several sets of Monte Carlo data, to which regular and irregular noise similar to the one

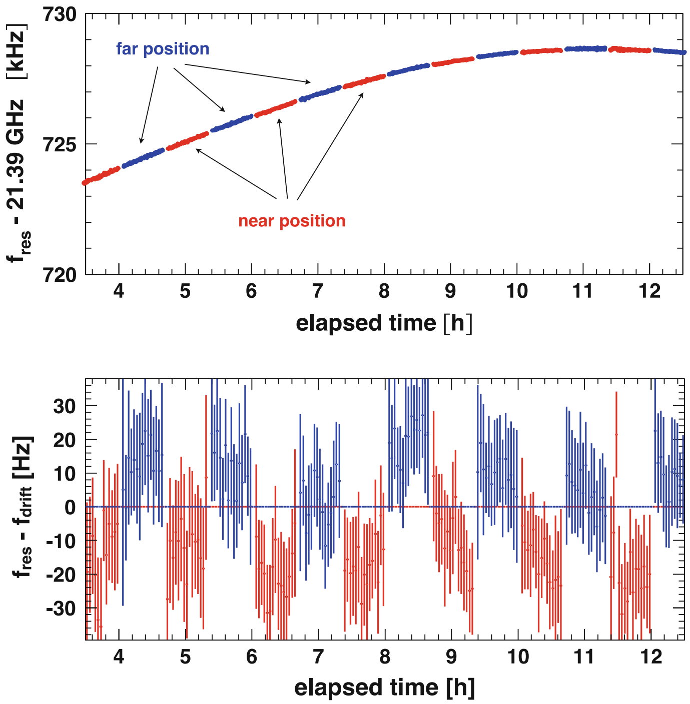

Fig. 3 Frequency as a function of time for a measurement with the 9.02 kg sphere, before (*above*) and after (*below*) a background drift subtraction. In the *upper* figure a large constant frequency offset has been subtracted

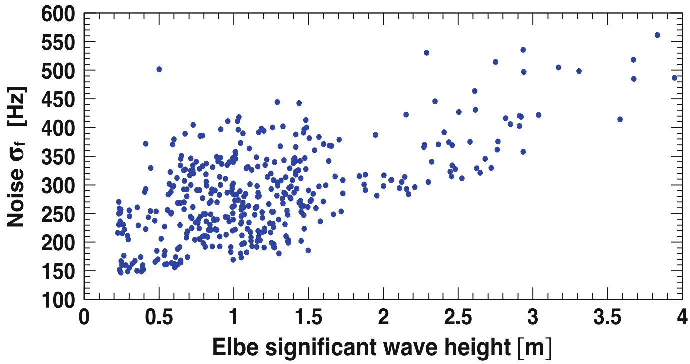

Fig. 4 The noise (rms) of a single frequency measurement as a function of the significant wave height at the mouth of the Elbe river

observed in the data had been added. All methods were able to reproduce the correct input signal within the uncertainties [20].

Method A1 was used as the central value and the other methods as consistency checks.

Table 1 Mean frequency shift Δf in Hz for the three field-masses

Method	Δf 1.0 (kg)	Δf 2.92 (kg)	Δf 9.02 (kg)
A1	2.06 ± 0.25	6.09 ± 0.24	18.65 ± 0.40
A2	2.00 ± 0.44	6.25 ± 0.32	19.12 ± 0.54
A3	2.06 ± 0.38	6.19 ± 0.34	18.53 ± 0.60
A4	2.94 ± 0.48	6.36 ± 0.31	19.30 ± 0.61
A5	1.72 ± 0.59	6.31 ± 0.36	20.09 ± 0.51
A6	1.60 ± 0.25	5.63 ± 0.23	17.00 ± 0.56

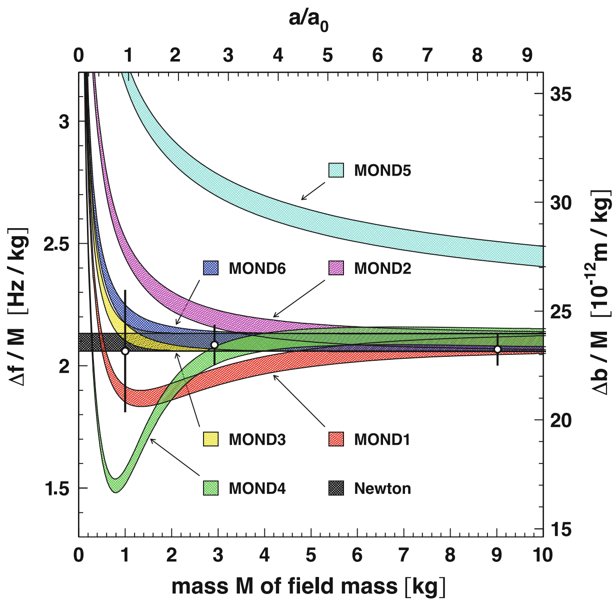

Fig. 5 Comparison of different versions of the MOND interpolation function with the measurements (MOND4 with $n = 10$). The values of the field-masses M are plotted against $\Delta f/M$, where Δf is the frequency shift. The *upper horizontal axis* indicates the value of the acceleration of one of the mirrors caused by the closer field-mass in the near position in units of the MOND acceleration a_0. The *vertical axis* on the right hand side shows the corresponding normalized change of distance between the mirrors

Assuming, that the potential effect due to MOND is negligible for the 9.02 kg field-masses, the acceleration being about $10\,a_0$, the gravitational constant G can be computed from the frequency shift Δf between the far and near positions of the 9.02 kg field-masses. The values of Δf given in Table 1 are the sum of the frequency shifts of the right and left field-masses. The frequency shift δf due to one field-mass is given by

$$\frac{\delta f}{f} = \frac{GMT_0^2 \Delta(1/r^2)}{4\pi^2 b} \tag{3}$$

Table 2 Systematic uncertainties contributing to the measurement of G

Source	Uncertainty in %
Pendulum frequency T_0^{-1}	0.66
Position of the field-masses r_n, r_f	1.48
Value of the field-masses M	0.11
Integration over the mass distribution of the mirrors	0.20
Distance between mirrors b	0.01

with

$$\Delta(1/r^2) = \left(1/r_n^2 - 1/(r_n + b)^2\right) - \left(1/r_f^2 - 1/(r_f + b)^2\right) \tag{4}$$

Here f is the frequency, M the field-mass, T_0 the pendulum period of the mirrors, b the distance between the two mirrors, r_n and r_f are the distances of the near and far position of the field-mass from the nearest mirror, respectively. Using method A1 as a reference value, and after a small correction taking account of the detailed shape of the mirrors [20], one obtains a value for G

$$G = (6.57 \pm 0.21 \pm 0.11) \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$$

where the first and dominating uncertainty is due to the uncertainty of the frequency shift. The second uncertainty is systematic, with the list of systematic error sources as given in the Table 2. The first three entries in the table follow directly from the corresponding measurement uncertainties, the fourth entry follows from the estimated accuracy of the integration over the mirrors, and the uncertainty of b was determined from an analysis of the mode spectrum of the resonator.

This value of G agrees with the world average [22] of $G = 6.67384(80) \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ within the uncertainties.

Predictions from MOND are not unambiguous. We assume, that the forces from each field-mass on each cavity, as calculated from the MOND formula, can be added linearly. With this assumption one can calculate the expected frequency shifts for the different field-masses and for different interpolation functions $\mu(x)$. Interpolation functions have been proposed inspired by data [5,23–25] or derived from the entropic force approach to gravitation [26,27]. The different interpolation functions are listed below, with a = acceleration due to MOND, a_N = Newton's acceleration, $x = a/a_0$ and $y = \sqrt{(a_N/a_0)}$:

MOND1 [5] $\quad \mu(x) = x/\sqrt{1 + x^2}$

MOND2 [23] $\quad \mu(x) = x/(1 + x)$

MOND3 [26] $\quad \mu(x) = 6x/\pi^2 \displaystyle\int_0^{\pi^2/6x} z/(e^z - 1)dz$

$$\text{MOND4 [25]} \quad a = a_0 y(1 - y^n)/(1 - y^{n-1})$$
$$\text{MOND5 [4]} \quad \mu(x) = (\sqrt{1+4x} - 1)/(\sqrt{1+4x} + 1)$$
$$\text{MOND6 [27]} \quad \mu(x) = (\sqrt{1+4x^2} - 1)/2x$$

Figure 5 shows a comparison of the measurements with the predictions of the five interpolation functions. For MOND4 the value $n = 10$ has been used [25]. The normalized frequency shift $\Delta f/M$ (where M is the mass of the field-mass) is plotted for the three field-masses. The error bars on the data points represent the uncertainty of the measured frequency shifts. The width of the bands for the interpolating functions shows the effect of the systematic uncertainties. If Newton's Law is valid, $\Delta f/M$ must be the same for all field-masses. The data are in good agreement with Newton's Law; the versions MOND1, MOND3 and MOND6 cannot be excluded. Version MOND5 due to Bekenstein's relativistic theory is clearly ruled out, within the assumptions made above (also ruled out by astrophysical observations [23]). The versions MOND2 and MOND4 are slightly disfavoured.

4 Conclusions

Newton's Law of Gravitation has been tested for small values of the acceleration, using a pair of pendulums to measure the gravitational attraction. The data cannot refute the MOND theory, but they have successfully probed Newton's Law down to the MOND acceleration $a_0 = 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$. The data are in good agreement with Newton's Law; the versions MOND1, MOND3 and MOND6 cannot be excluded. Version MOND5 due to Bekenstein's relativistic theory is clearly ruled out, within the assumptions made above (also ruled out by astrophysical observations [23]). The versions MOND2 and MOND4 are slightly disfavoured.

The accuracy of the measurements will be improved by moving the experiment to an underground location with a more quiet environment and by implementing a significantly improved mechanical support structure which allows better control of all parameters that currently are dominating the systematic uncertainties.

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